

Example of Regex, NFA, DFA

Prof. James L. Frankel
Harvard University

Version of 5:55 PM 30-Mar-2022
Copyright © 2022, 2020 James L. Frankel. All rights reserved.

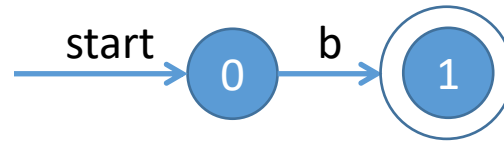
Initial Regex

- Regex: $ba^*b(a|b)a$
- Expressed in English
 - One “b” followed by zero or more “a”s followed by one “b” followed by either “a” or “b” followed by “a”

Construction of each NFA: expression a in Σ

Regex: **b**a*b(a|b)a

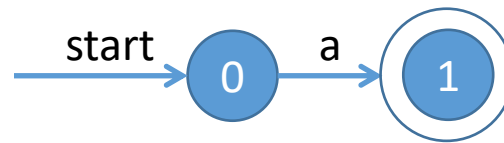
Regex for: b



Construction of each NFA: expression a in Σ

Regex: $b\mathbf{a}^*b(a|b)a$

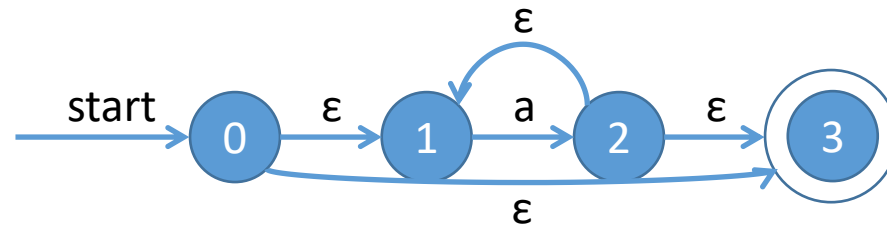
Regex for: a



Construction of each NFA: expression with closure (that is, *)

Regex: $b\mathbf{a}^*b(a|b)a$

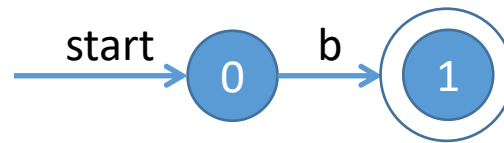
Regex for: a^*



Construction of each NFA: expression a in Σ

Regex: $ba^*b(a|b)a$

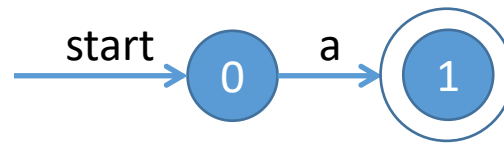
Regex for: b



Construction of each NFA: expression a in Σ

Regex: $ba^*b(a|b)a$

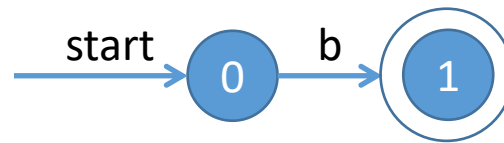
Regex for: a



Construction of each NFA: expression a in Σ

Regex: $ba^*b(a|b)a$

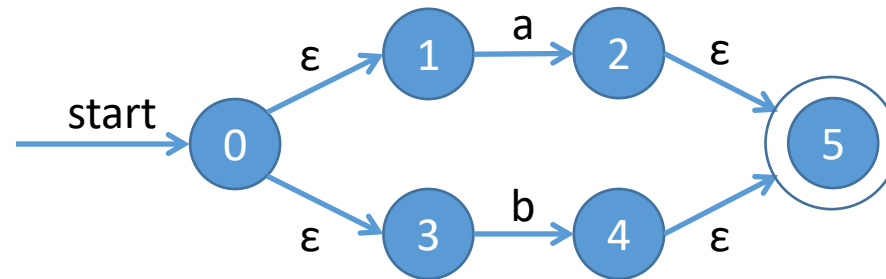
Regex for: b



Construction of each NFA: expression with union (that is, |)

Regex: $ba^*b(a|b)a$

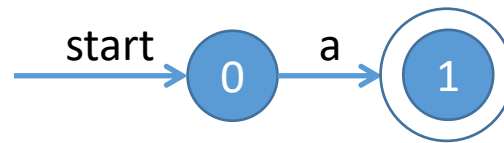
Regex for: $(a|b)$



Construction of each NFA: expression a in Σ

Regex: $ba^*b(a|b)a$

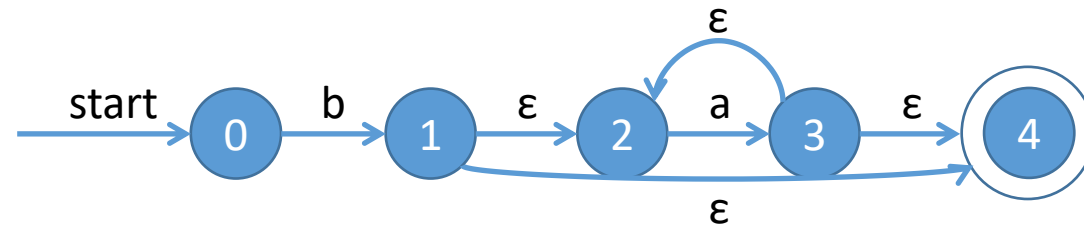
Regex for: a



Construction of each NFA: expression with concatenation

Regex: **ba***b(a|b)a

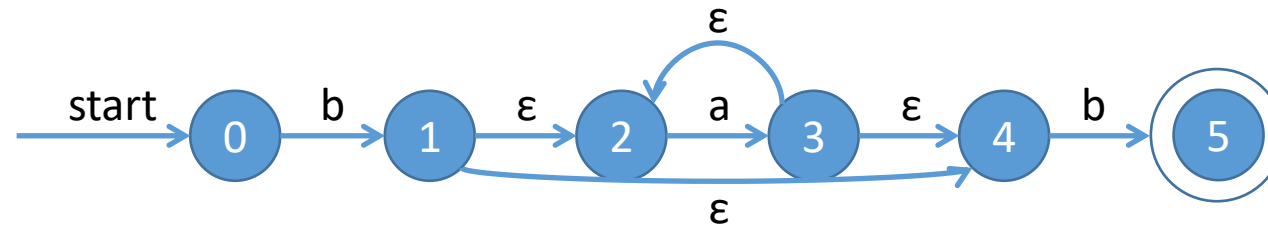
Regex for: **ba***



Construction of each NFA: expression with concatenation

Regex: **ba*b**(a|b)a

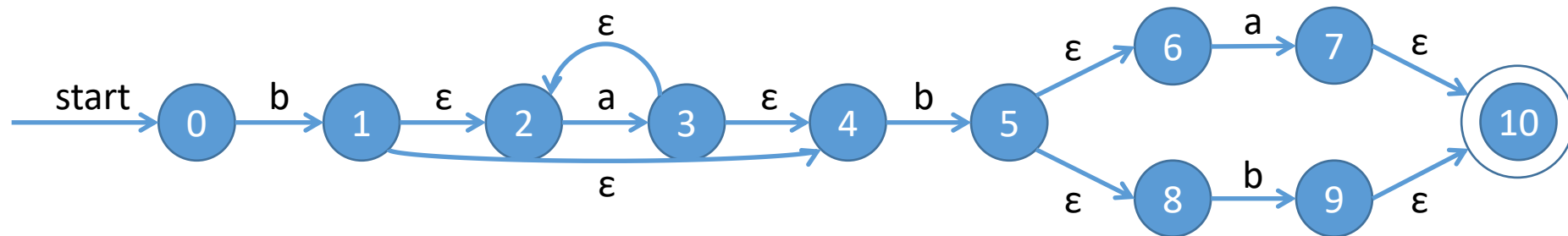
Regex for: **ba*b**



Construction of each NFA: expression with concatenation

Regex: **ba*b(a|b)a**

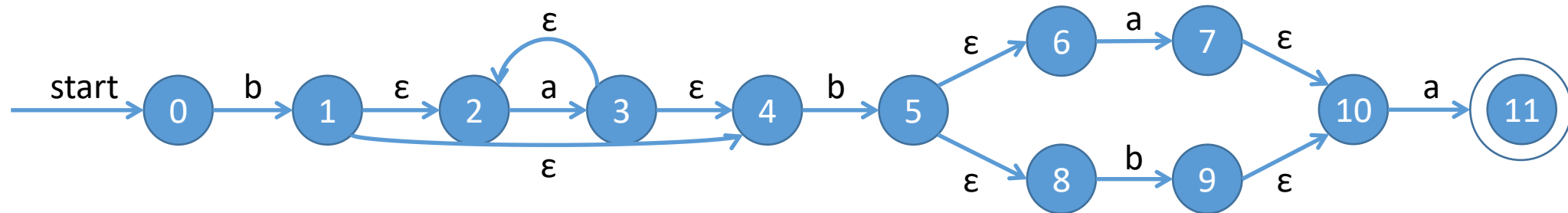
Regex for: $ba^*b(a|b)$



Construction of each NFA: expression with concatenation

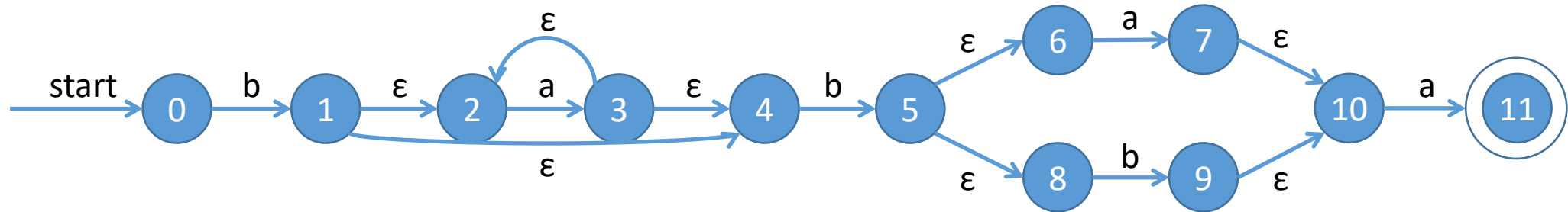
Regex: **ba*b(a|b)a**

Regex for: $ba^*b(a|b)a$



Final Complete NFA Constructed from Regex

Regex: $ba^*b(a|b)a$



Subset Construction of DFA from NFA

Regex: $ba^*b(a|b)a$

Unmarked state is indicated by red

$$Dstates = \{ \epsilon\text{-closure}(s_0) \} = \{ \{0\} \}$$

$$Dstates = \{ \{0\}, \epsilon\text{-closure}(\text{move}(\{0\}, b)) \} = \\ \{ \{0\}, \epsilon\text{-closure}(\{1\}) \} = \\ \{ \{0\}, \{1, 2, 4\} \}$$

$$Dtran[\{0\}, b] = \{1, 2, 4\}$$

Subset Construction of DFA from NFA

Regex: $ba^*b(a|b)a$

$Dstates = \{ \{0\}, \{1, 2, 4\}, \epsilon\text{-closure}(\text{move}(\{1, 2, 4\}, a)),$
 $\epsilon\text{-closure}(\text{move}(\{1, 2, 4\}, b)) \} =$
 $\{ \{0\}, \{1, 2, 4\}, \epsilon\text{-closure}(\{3\}), \epsilon\text{-closure}(\{5\}) \} =$
 $\{ \{0\}, \{1, 2, 4\}, \{2, 3, 4\}, \{5, 6, 8\} \}$

$Dtran[\{1, 2, 4\}, a] = \{2, 3, 4\}$

$Dtran[\{1, 2, 4\}, b] = \{5, 6, 8\}$

Subset Construction of DFA from NFA

Regex: $ba^*b(a|b)a$

Dstates = { {0}, {1, 2, 4}, {2, 3, 4}, {5, 6, 8},
 ϵ -closure(move({2, 3, 4}, a)), ϵ -closure(move({2, 3, 4}, b)),
 ϵ -closure(move({5, 6, 8}, a)), ϵ -closure(move({5, 6, 8}, b)) } =
{ {0}, {1, 2, 4}, {2, 3, 4}, {5, 6, 8},
 ϵ -closure({3}), ϵ -closure({5}),
 ϵ -closure(move({5, 6, 8}, a)), ϵ -closure(move({5, 6, 8}, b)) } =
{ {0}, {1, 2, 4}, {2, 3, 4}, {5, 6, 8},
 ϵ -closure(move({5, 6, 8}, a)), ϵ -closure(move({5, 6, 8}, b)) }

Dtran[{2, 3, 4}, a] = {2, 3, 4}

Dtran[{2, 3, 4}, b] = {5, 6, 8}

Subset Construction of DFA from NFA

Regex: $ba^*b(a|b)a$

Dstates = { {0}, {1, 2, 4}, {2, 3, 4}, {5, 6, 8},
 ϵ -closure(move({5, 6, 8}, a)),
 ϵ -closure(move({5, 6, 8}, b)) } =
{ {0}, {1, 2, 4}, {2, 3, 4}, {5, 6, 8},
 ϵ -closure({7}), ϵ -closure({9}) } =
{ {0}, {1, 2, 4}, {2, 3, 4}, {5, 6, 8}, {7, 10}, {9, 10} }

Dtran[{5, 6, 8}, a] = {7, 10}

Dtran[{5, 6, 8}, b] = {9, 10}

Subset Construction of DFA from NFA

Regex: $ba^*b(a|b)a$

Dstates = { {0}, {1, 2, 4}, {2, 3, 4}, {5, 6, 8}, {7, 10}, {9, 10},
 ϵ -closure(move({7, 10}, a)), ϵ -closure(move({7, 10}, b)),
 ϵ -closure(move({9, 10}, a)), ϵ -closure(move({9, 10}, b)) } =
{ {0}, {1, 2, 4}, {2, 3, 4}, {5, 6, 8}, {7, 10}, {9, 10},
 ϵ -closure({11}),
 ϵ -closure(move({9, 10}, a)), ϵ -closure(move({9, 10}, b)) } =
{ {0}, {1, 2, 4}, {2, 3, 4}, {5, 6, 8}, {7, 10}, {9, 10}, {11},
 ϵ -closure(move({9, 10}, a)), ϵ -closure(move({9, 10}, b)) }

Dtran[{7, 10}, a] = {11}

Subset Construction of DFA from NFA

Regex: $ba^*b(a|b)a$

Dstates = { {0}, {1, 2, 4}, {2, 3, 4}, {5, 6, 8}, {7, 10}, {9, 10}, {11},
 ϵ -closure(move({9, 10}, a)), ϵ -closure(move({9, 10}, b)) } =
{ {0}, {1, 2, 4}, {2, 3, 4}, {5, 6, 8}, {7, 10}, {9, 10}, {11},
 ϵ -closure(move({9, 10}, b)) } =
{ {0}, {1, 2, 4}, {2, 3, 4}, {5, 6, 8}, {7, 10}, {9, 10}, {11} }

Dtran[{9, 10}, a] = {11}

Subset Construction of DFA from NFA

Regex: $ba^*b(a|b)a$

Dstates = { {0}, {1, 2, 4}, {2, 3, 4}, {5, 6, 8}, {7, 10}, {9, 10}, {11} } =
{ {0}, {1, 2, 4}, {2, 3, 4}, {5, 6, 8}, {7, 10}, {9, 10}, {11},
 ϵ -closure(move({11}, a)), ϵ -closure(move({11}, b)) } =
{ {0}, {1, 2, 4}, {2, 3, 4}, {5, 6, 8}, {7, 10}, {9, 10}, {11} }

Subset Construction of DFA from NFA: Resulting Dstates and Dtran

Dstates = { {0}, {1, 2, 4}, {2, 3, 4}, {5, 6, 8}, {7, 10}, {9, 10}, {11} }

Dtran =

D State	Input Symbol	New D State
{0}	b	{1, 2, 4}
{1, 2, 4}	a	{2, 3, 4}
{1, 2, 4}	b	{5, 6, 8}
{2, 3, 4}	a	{2, 3, 4}
{2, 3, 4}	b	{5, 6, 8}
{5, 6, 8}	a	{7, 10}
{5, 6, 8}	b	{9, 10}
{7, 10}	a	{11}
{9, 10}	a	{11}

Subset Construction of DFA from NFA: Renamed Dstates and Dtran

Dstates = { A, B, {2, 3, 4}, {5, 6, 8}, {7, 10}, {9, 10}, {11} }

Dtran =

D State	Input Symbol	New D State
A	b	B
B	a	C
B	b	D
C	a	C
C	b	D
D	a	E
D	b	F
E	a	G
F	a	G

Resultant DFA

- Expressed in English
 - One “b” followed by zero or more “a”s followed by one “b” followed by either “a” or “b” followed by “a”

